'Creating Measures' Square-ness Task - Example #1 (solutions)

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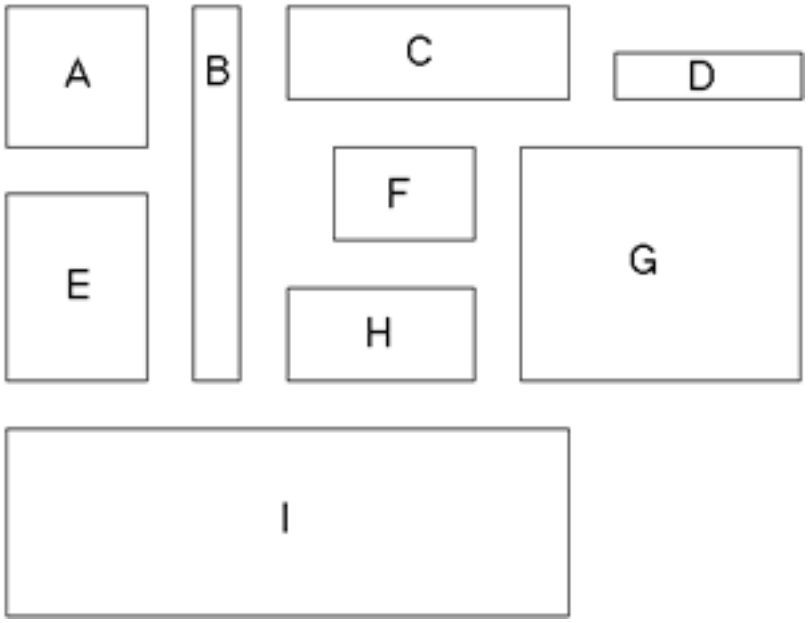
This problem gives you the chance to:

- criticise a given measure for the concept of "square-ness"
- invent your own ways of measuring this concept
- examine the advantages and disadvantages of different methods.



Warm-up

Use visual judgements to answer the warm-up questions.
Which rectangle *looks* the most square?
Which rectangle *looks* least square?

Without measuring anything, put the rectangles in order of "square-ness."

Comment:

This first question is simply intended to orientate the students to the task in hand. It may be used as a class discussion.

1. Someone has suggested that a good measure of "square-ness" is to calculate the difference:

Longest side - shortest side

for each rectangle. Use this definition to put the rectangles in order of "square-ness." Show all your work.

Solution:
Using the measure 'Longest side - shortest side', the "square-ness" of each rectangle is given in the table below (using centimeters as the unit).

Rectangle	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
Dimensions (cm)	3 x 3	1 x 8	6 x 2	4 x 1	3 x 4	3 x 2	6 x 5	4 x 2	12 x 4
Square-ness (cm)	0	7	4	3	1	1	1	2	8

Using this measure, the rectangles in order from most to least square are:
A, E and F and G (tie), H, D, C, B, I.

2. Using your results, give one good reason why **Longest side - shortest side** is not a suitable measure for "square-ness."

Solution:
The above measure is unsatisfactory because:

- It gives no indication of the overall 'proportions'. (E, F and G under this definition have the same square-ness yet are clearly different in shape, while C and I are similar in shape but give different square-ness measures).

- It is dependent on the units used. If we use inches instead of centimetres we get a different "square-ness" measure.

3. Invent a different way of measuring "square-ness." Describe your method carefully below:

Solution:
There are many other ways of measuring "square-ness." Students might, for example, propose using:

a) The ratio longest side/shortest side;

b) The largest angle between the diagonals of the rectangle;

c) The ratio of perimeter/area.

a) and b) seem equally sensible. c), however, suffers the same problem as before. As it is not dimensionless, an enlargement of a rectangle will result in a different value for its "square-ness."

If, however, we use

d) the ratio $(\text{perimeter})^2/\text{area}$

then we would have a suitable, dimensionless measure.

4. Place the rectangles in order of "square-ness" using your method. Show all your work.

Solution:
Whichever measure we now use (a), (b) or (d), we obtain the same order for the rectangles. In order of "square-ness" they are:

A (most square), G, E, F, H, C and I (tie), D, B (least square).

Rectangle	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
Dimensions (cm)	3 x 3	1 x 8	6 x 2	4 x 1	3 x 4	3 x 2	6 x 5	4 x 2	12 x 4
Ratio: Longest ÷ Shortest	1	8	3	4	1.3	1.5	1.2	2	3
Largest angle between diagonals	90˚	166˚	143˚	152˚	106˚	113˚	100˚	127˚	143˚
Ratio: Perimeter^2 ÷ area	16	40.5	21.3	25	16.3	16.7	16.1	18	21.3

5. Do you think your measure is a good way of measuring "square-ness?" Explain your reasoning carefully.

Solution:
Here we would like students to review their results critically and decide whether the results from their measurements accord with their intuitions.

6. Find a different way of measuring "square-ness."
Compare the two methods you invented. Which is best? Why?

Solution:
This question provides an opportunity for students to look for an alternative measure.