



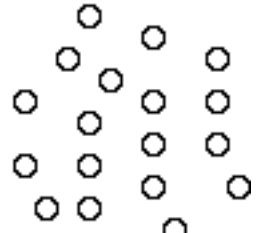
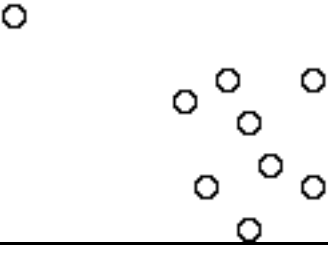
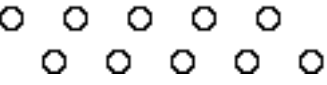
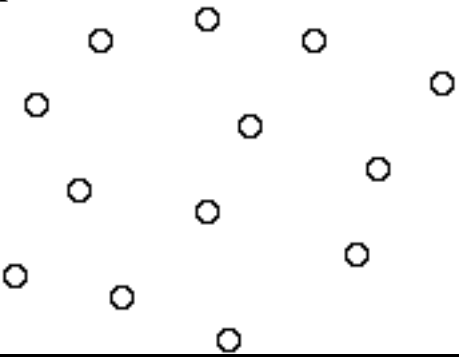
'Creating Measures' Crowded-ness Task - Example #4 (solutions)

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This problem gives you the chance to:

- criticise a given measure for the concept of "crowded-ness"
- invent your own way of measuring this concept and use it effectively

Group A 	Group B 
Group C 	Group D 
Group E 	Group F 

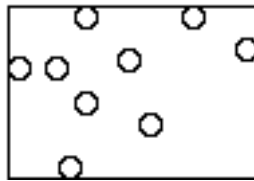
Warm-up

In your opinion, which group *looks* the least "crowded"?
Which *looks* the most "crowded"?
Put the groups in order of "crowdedness", just by looking.

Comment:

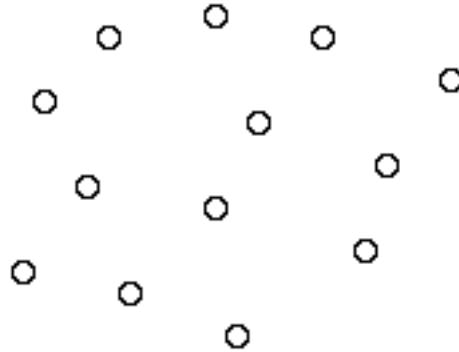
Intuitively, most people would consider the least crowded to be F, and the most crowded group to be C and, they would give the following rough order for "crowdedness": F, B, A, C. The remaining groups are quite 'odd' in that D seems to be distributed less evenly and E seems to be distributed somewhat linearly. There should therefore be some disagreement about these points. Intuitively, "crowdedness" gives the feeling of "closeness" or "confined-ness"; thus it may mean the proximity of one person to another or the denseness of a collection of people in a defined space.

1. Someone has suggested that "crowdedness" may be defined as the **area** of a box that will *just* enclose all the people in a cluster. So for group A:



"Crowded-ness" = 7.26 m^2

Use this method to calculate the crowdedness of group F:



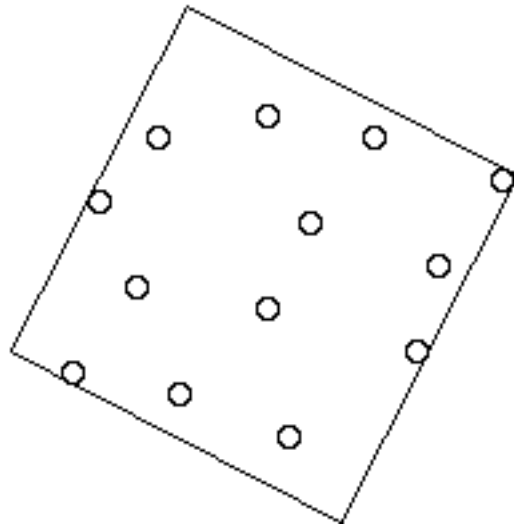
Solution:

The solution for group F gives a rectangle which is approximately 27 m^2 in area.

2. Write down **at least two** reasons why this is not a good way of measuring "crowdedness".

Solution:

This area is not a good measure for "crowdedness," because it takes no account of the number of people in the rectangle. Also, there is more than one rectangle that *just* touches the extreme people, giving a different area. For example:



3. Describe a better method for measuring "crowdedness". Explain why it is better.

Solution:

A better solution would be one in which we

- provide an unambiguous method for defining the area occupied.
- divide the number of people by this area, to give a 'density' (people per unit area) definition.

For example, we could define "crowdedness" as:

- a) The number of people divided by the area of the smallest (convex) polygon that can contain the set of people.
- b) The number of people divided by the area of the smallest circle that will surround the people.

(Other possible measures might use distances between people, for instance: the distance between the two most remote people, or the sum of the distances between pairs of people.)

4. Use your method to give a numerical value to the "crowdedness" of Groups A to F. Use your method to put the groups in order of "crowdedness".

Solution:

Next, having chosen a measure of compactness, the student must show its application by computing numerical compactness values for the given groups.

Using the definition: area of smallest circle that will surround the group divided by the number of people within it, I obtained the following results (units are in people per square meter).

Group	A	B	C	D	E	F
Crowded-ness	1.06	0.80	1.59	0.45	0.64	0.40

Thus the order from the least crowded to most crowded, using this definition is:
F, D, E, B, A, C.