'Plausible Estimation' Estimates for the USA Tasks - Set #4 (solutions)

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The aim of this assessment is to provide the opportunity for you to:

- develop a chain of reasoning that will enable you to make reasonable estimates of facts about the USA from population data
- communicate the assumptions upon which your estimate is based.

Try to estimate reasonable answers to each of the following questions.

Describe carefully at each stage any assumptions you make.

Show, step by step how you arrive at your estimate.

Each estimate should use the following fact:

The population of the USA is approximately 270 million.

1. Babies

How many babies are born in the USA in each minute?

Assumptions

- The population of the USA is approximately 270 million.
- Assume that the average life span is approximately 70 years.

- Assume the population is fairly stable.
- Assume that the distribution of the population is such that 1/70th of the population is 1 year old.

Reasoning

Then we may assume that 270/70 million babies are born each year.

$270 \times 10^6/70 = 3.9 \times 10^6$ babies per year

$3.9 \times 10^6/(365 \times 24 \times 60)$ = babies per minute

Answer: 7.4 babies per minute.
(Verification: In 1996, 3,915,000 babies were born.)

2. Teachers

How many K-8 Elementary School teachers are there in the USA?

Assumptions

- The population of the USA is approximately 270 million.
- Approximately 1/8 of the population is in years K through 8.
  (If life span is approx 75 years and 9 of these are K-8, but fewer people are older)
- Assume an average of 25 children per class.
- Assume all teachers are working full-time.

Reasoning

Then there will be

$270 \times 10^6/(8 \times 25) = 1.35 \times 10^6$

Answer: 1.35 million Elementary teachers.

(Verification: This estimate is a bit low. There are actually approximately 1.9 million.)

3. Newspapers

How much is spent on newspapers in the USA each year?

Assumptions

- The population of the USA is approximately 270 million.
- Assume that there are 3 people per household, on average.
- Assume that each household buys 1 paper per day.
- Assume that average cost of newspaper is $0.60.

Reasoning

There are thus approximately

$270 \times 10^6/3 = 90 \times 10^6$

Each year they will spend 90 x 106 x 365 x 0.60 = 20 x 109 dollars on newspaper.

Answer: $20 billion.
(Verification: This estimate is a bit low. Actual figure is about 26 billion dollars.)

4. Cars

How many cars are bought each year in the USA?

Assumptions

- The population of the USA is approximately 270 million.
- Assume one car is owned per household.
- Assume that there are 3 people per household.
- Assume that life expectancy of a car is about 10 years.
- Assume that 1/10 of cars are replaced each year by new ones.

Reasoning

Then there are approximately

$270 \times 10^6/3 = 90 \times 10^6$ cars on the road in the US.

If we divide this figure by 10, we can see that approximately 9 million cars are bought each year.

Answer: 9 million.
(Verification: In 1997, the actual figure was approximately 8 million.)

5. Dentists

How many dentists are there in the USA?

Assumptions

- The population of the USA is approximately 270 million.
- Each member of population sees dentist on average twice per year.
- Each consultation lasts about 20 minutes.
- Dentist sees patients for about 6 hours per day.
- Dentist works for 45 weeks per year.

Reasoning

Then, dentist can have 45 x 5 x 6 x 3 = 4,050 consultations per year.

Number of consultations required = 540 million

540 million ÷ 4,050 = 130 thousand

Answer: About 130 thousand dentists are required.
(Verification: There are 160 thousand active dentists, or one dentist for every 1,600 of the population.)